NUMERICAL EXPERIMENT ON SELF-FOCUSING OF ELECTROMAGNETIC WAVES IN A NONLINEAR MEDIUM

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Results are presented of a numerical experiment on the propagation of broad axially symmetric wave beams in a weakly nonlinear medium. Cases of cubical nonlinearity and nonlinearity with saturation are examined.

As is known, the propagation of sufficiently broad axially symmetric wave beams in a weakly nonlinear medium without absorption with the dielectric constant

$$\varepsilon = \varepsilon_0 [1 + f(|u|^2)], \quad f(|u|^2) \ll 1 \tag{1}$$

is described by the parabolic equation [1, 2]

$$2i \frac{\partial u}{\partial z'} = \frac{\partial^2 u}{\partial r'^2} + \frac{1}{r'} \frac{\partial u}{\partial r'} + k^2 f(|u|^2) u \qquad \left(k = \frac{\omega \sqrt{\varepsilon_0}}{c}\right) \tag{2}$$

Here r' is the radial coordinate, z' the axial coordinate, c the speed of light, ω the frequency, and u the electric field intensity.

In the dimensionless variables **r** = **kr'**, (1) takes the form

$$2i \frac{\partial u}{\partial z} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + f(|u|^2) u \tag{3}$$

The difficulties in analytic study of (3) make necessary its numerical integration. In the present paper this integration is carried out for a medium with cubical nonlinearity

$$f(|u|^2) = \sigma |u|^2 \quad (\sigma > 0) \tag{4}$$

and for a medium with saturated nonlinearity of the form

$$f(|u|^2) = \sigma \varkappa^{-1} (1 - \exp(-\varkappa |u|^2)) \qquad (\sigma, \varkappa > 0) \tag{5}$$

The choice of (5) is a result of the fact that for sufficiently small $|u|$ the medium (5) can be considered cubical, and for $|u| \gg 1$ the parabolic equation (3) becomes linear.

As the initial condition for (3) we take the Gaussian distribution

$$u(r, 0) = \exp(-r^2 / l^2) \tag{6}$$

where l is the characteristic width of the initial beam.

The natural boundary conditions for (3) have the form [1, 3]

$$\frac{\partial u}{\partial r}(0, z) = 0, \qquad u(\infty, z) = 0 \tag{7}$$

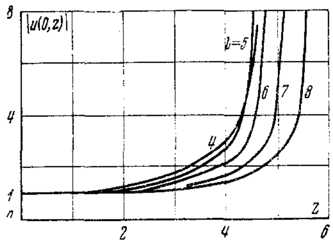

Fig. 1

Novisibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 10, No. 6, pp. 20-22, November-December, 1969. Original article submitted March 5, 1969.

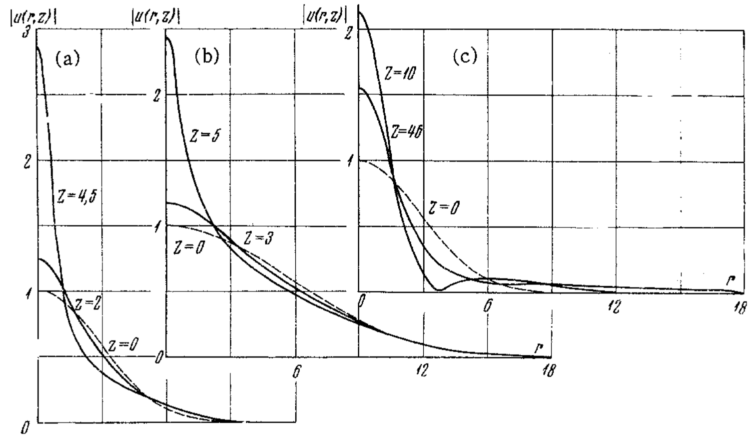

Fig. 2

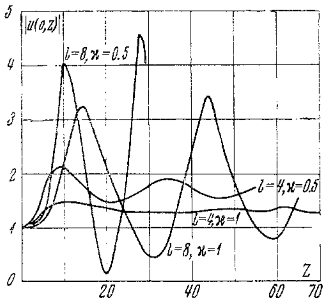

Fig. 3

We note that from convergence of the energy integral

$$P = \int_0^\infty |u|^2 r dr \tag{8}$$

it follows that $|u|$ decreases faster than r^{-1}.

For the boundary condition at infinity we use an approximation of the form

$$\partial u\ (R,\ z)\ /\ \partial r = \alpha\ (R)\ u\ (R,\ z) \tag{9}$$

where R is the end point of the numerical integration interval. To obtain $\alpha(R)$ we use the method developed in [4] for linear differential equations. The possibility of using the algorithm of [4] in the present case is associated with the fact that the nonlinear term $f(|u|^2)u$ decreases at infinity at least as r^{-3}. Equation (3) is approximated by an implicit two-layer finite-difference scheme of second order in r and first order in z [5]. The resulting system of algebraic equations for the unknown grid functions on the different z layers was solved by the pivotal method. The calculation was made on a BESM-6 computer in the band $0 \leq r \leq R$ and was terminated upon reaching a prespecified value of z. The quantity R was taken in the range from $5l$ to $10l$. In practice the calculations using this algorithm were stable for any r and z steps.

To check the correctness of the calcuation we used conservation of the energy integral P, which can be represented in a form convenient for numerical realization

$$P = P_1 + P_2,\ P_1 = \int_0^R |u|^2 r dr, \qquad P_2 = R\,\mathrm{Im}\int_0^z \left(u\ \frac{\partial u}{\partial r}\right)_{r=R} dz \tag{10}$$

Formula (10) is obtained easily if we multiply both sides of (3) by u and apply Green's formula to the imaginary part of the resulting expression.

In [1, 2] it is shown that in a cubical medium with beam power P exceeding some critical value P_*, the beam self-constricts (collapses) to a point on the z axis. Numerical calculations [1] show that the critical power corresponds to $l_* \approx 2.73$. For $l > l_*$ the beam intensity on the axis increases. The variation of the field amplitude along the beam axis for various $l > l_*$ is shown in Fig. 1.

It turns out that about 20-30% of all the light pulse energy is concentrated in the collapsed part of the beam. The formation of side peaks on the $|u(r, z)|$ profile for fixed z, indicated in [3], was not observed. The characteristic amplitude profiles for different z are shown in Fig. 2a for $l = 4$ and in Fig. 2b for $l = 8$.

Collapse does not occur in a medium with saturation of the nonlinearity for light beam power P exceeding the critical value P_*. The amplitude on the axis oscillates and with increase of z changes quite complexly (Fig. 3). We see from Fig. 3 that with increase of l for given z, the maximal value of the light beam amplitude on the axis increases. The range of the $|u(0, z)|$ oscillations decreases with increase of $\varkappa$ and the amplitude on the axis approaches some stationary state. Figure 2c shows the $|u(0, z)|$ profiles for $l = 4$, $\varkappa = 0.5$.

In the calculation process the errors associated with the error of approximation of the initial equation (3) accumulate. This leads to a slight change of the energy integral calculated using (11). The maximal beam penetration depth in Figs. 1 and 3 corresponds to an energy "change" of 3% with respect to the initial value.

These calculations are applicable to time intervals after the appearance of the light pulse which are less than the time necessary for the manifestation of thermal and striction effects.

The authors wish to thank V. I. Karpman for valuable discussions and his interest in this study.

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